

## 5. cvičení - Limity posloupností 3

**Příklad 1.** Limita posloupnosti zkoušková obtížnost:

- (a)  $\lim_{n \rightarrow \infty} \frac{2^n \sqrt[3]{n^3+n^2} - n \sqrt[3]{8n+1}}{\sqrt[n]{2n^2+1}}$
- (b)  $\lim_{n \rightarrow \infty} \left( \sqrt{n^2 + \sqrt{n} + 1} - \sqrt{n^2 + 2\sqrt{n} + 3} \right) \frac{\sqrt[n]{n+n^n}}{[\sqrt{n+2}]}$
- (c)  $\lim_{n \rightarrow \infty} \left( \sqrt[3]{2n+a} - \sqrt[3]{2n+b} \right) \cdot \sqrt[3]{(n+1)(3n+2)}, a, b \in \mathbb{R}, a > 0, b > 0$
- (d)  $\lim_{n \rightarrow \infty} \frac{(n^2+1)^{100} - (n+2)^{200} + 400n^{199}}{1+2+\dots+n^{99}}$
- (e)  $\lim_{n \rightarrow \infty} n^3 \left( \sqrt[3]{n! + 3^n} - \sqrt[3]{n! + 2^n} \right)$
- (f)  $\lim_{n \rightarrow \infty} \frac{(n+\sin n)^7 - (n+\sqrt{n})^7}{n^2 \sqrt{n^7+7}} \cdot \arctan \frac{1}{n},$  víte-li že  $\lim_{n \rightarrow \infty} n \cdot \arctan \frac{1}{n} = 1$
- (g)  $\lim_{n \rightarrow \infty} \left( \sqrt[3]{3^n + 2^n} - \sqrt[3]{3^n + n} \right) \cdot \left( \frac{3^n + 2^n}{3^n + n} + (-1)^n n^2 \right)$

**Příklad 2.** Limita posloupnosti zkoušková obtížnost:

- (a)  $\lim_{n \rightarrow \infty} \frac{\lfloor \sqrt[3]{n^3+1} \rfloor - \lfloor \sqrt[3]{n^3-1} \rfloor}{\sqrt[n]{1+2^n+\dots+n^n}}$
- (b)  $\lim_{n \rightarrow \infty} \lfloor \sqrt[4]{n^4 + 4n^3} - n \rfloor$
- (c)  $\lim_{n \rightarrow \infty} \frac{\sqrt{n+\sin^2 n} - \sqrt{n-\cos^2 n}}{\sqrt{n+1} - \sqrt{n-1}}$
- (d)  $\lim_{n \rightarrow \infty} \frac{(n+7)^{50} - (n^2+1)^{25}}{\sqrt{n^{100}+n^{99}-1} - \sqrt{n^{100}+2n^{99}+1}}$
- (e)  $\lim_{n \rightarrow \infty} \left( \sqrt[3]{n^4 + \sqrt[3]{n}} - \sqrt[3]{n^4} \right) (\lfloor \sqrt[3]{n+1} \rfloor + \lfloor 2\sqrt[3]{n-1} \rfloor + \dots + \lfloor n \cdot \sqrt[3]{n + (-1)^{n+1}} \rfloor)$
- (f)  $\lim_{n \rightarrow \infty} \frac{n\sqrt{n} \sqrt[n]{(n+1)^n + n^{n+1}}}{\lfloor \sqrt{n} \rfloor + \lfloor 2\sqrt{n} \rfloor + \dots + \lfloor \sqrt{n} \rfloor}$
- (g)  $\lim_{x \rightarrow \infty} \frac{\sqrt{n^2+n} - \sqrt[4]{n^4+n^3}}{\sqrt{n^2+3n} - \sqrt[3]{n^3+2n}}$

